4.1 Remind me of inverse functions. Can I use the chain rule?

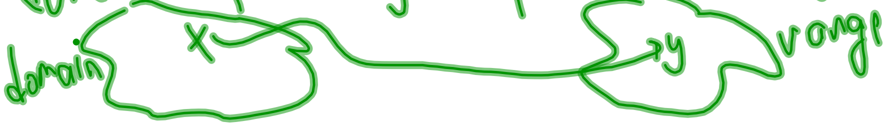
2014-11-10 day 52
fire drill spot: 18C

Recall

A relation f(x) is a function if, for every x in the domain of f(x), there is only 1 value of y that "corresponds" to that x.

Graphically, a set [['locus']] of graphed points is a function if it passes the 'vertical line' Test [[every vertical line intersects the graph in 0 or 1 point]].

We can also think of a function as a mapping from the domain to the range, where each value in the domain maps to only one point in the range



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Inverse relations and functions

An inverse relation $f^{-1}(x)$ is a function if for every y in the range of $f(x)$ [ie. the domain of $f^{-1}(x)$], there is only one x in the domain of $f(x)$ [ie. the range of $f^{-1}(x)$]

- Captures the idea of "undoes" or "backwards"
- Graphically, the graph of $f^{-1}(x)$ is just the reflection across $y=x$ of the graph of $f(x)$.
- If you are looking at the set of ordered pairs in $f(x)$, you can "switch" the x- and y-coordinates to get the inverse.

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A definition of inverse functions . . .

$f(x)$ and $g(x)$ are inverse functions iff:

a) $f(g(x)) = x$ for every x in the domain of $g(x)$

b) $g(f(x)) = x$ for every x in the domain of $f(x)$.

In this case, we write $g(x) = f^{-1}(x)$

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If $f(f^{-1}(x)) = x$, then taking the derivative (using the chain rule)

$$f'(f^{-1}(x)) \cdot (f^{-1})'(x) = 1$$

and so....

$$f'(f^{-1}(x)) = \frac{1}{(f^{-1})'(x)}$$

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Practically...

e^x and $\ln x$ are inverse functions

so

$\ln(e^x) = x$ for every x

taking the derivative...

$\frac{d}{dx}(\ln x)\Big|_{x=e^x} \cdot \frac{d}{dx}(e^x) = 1$

$\frac{d}{dx}(\ln x)\Big|_{(x=e^x)} \cdot e^x = 1$

$\frac{d}{dx}(\ln x)\Big|_{x=e^x} = \frac{1}{e^x}$

$\frac{d}{dx}(x^n) = n x^{n-1}$

new primitive rule

$\frac{d}{dx}(e^x) = e^x$

and so

$\frac{d}{dx}(b^x) = b^x(\ln b)$

this is NOT the product rule.

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Practically...

e^x and $\ln x$ are inverse functions

so ... another way

if y = the inverse function of e^x ...

$e^y = x$

$\frac{d}{dx}(e^y) = 1$

$e^y \cdot \frac{dy}{dx} = 1$

so $\frac{dy}{dx} = \frac{1}{e^y}$

Now .... if y = inverse of e^x = $\ln x$

$\frac{d}{dx}(\ln x) = \frac{1}{e^{\ln x}} = \frac{1}{x}$ ☆

$\frac{d}{dx}(x^n) = n x^{n-1}$

new primitive rule

$\frac{d}{dx}(e^x) = e^x$

and so

$\frac{d}{dx}(b^x) = b^x (\ln b)$

this is NOT the product rule.

new primitive rule

$$\boxed{\frac{d}{dx}(\ln x) = \frac{1}{x}}$$

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$$\frac{d}{dx}(e^x) = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h\to 0} \frac{e^x e^h - e^x(1)}{h}$$

$$= \lim_{h\to 0} \frac{e^x(e^h - 1)}{h} = e^x \underbrace{\lim_{h\to 0} \frac{e^h - 1}{h}}_{\text{this I don't remember how to show...}}$$

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$$\tan(\tan^{-1}(x)) = x$$

take the derivative . . . . .

$$\frac{d}{dx}(\tan x)\Big|_{x=\tan^{-1}(x)} \cdot \frac{d}{dx}(\tan^{-1}(x)) = 1$$

$$\underline{\sec^2 x\Big|_{x=\tan^{-1}(x)}} \cdot \frac{d}{dx}(\tan^{-1}(x)) = 1$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{\sec^2(\tan^{-1}(x))} = \cos^2(\tan^{-1}(x))$$

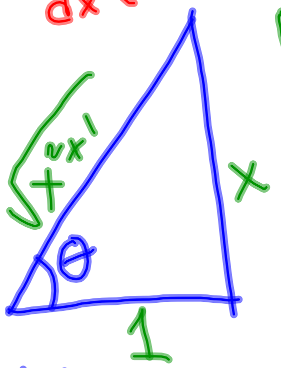
$$\Downarrow$$

$$\cos^2(\theta)$$

$$\Downarrow$$

$$\left(\frac{1}{\sqrt{x^2+1}}\right)^2$$

$$= \frac{1}{x^2+1}$$



$$\cos\theta = \frac{1}{\sqrt{x^2+1}}$$

recall that

$\tan^{-1}(x)$ = the angle (θ) whose tangent is x.

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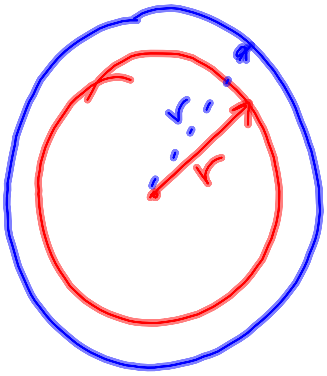
3.7 13



always true

$$\frac{dA}{dt} = 6\,\frac{mi^2}{hr}$$

specific instant

$$A = 9\ mi^2$$

$$A = \pi r^2 = 9\ mi^2$$

$$r^2 = \frac{9}{\pi}$$

$$r = \sqrt{\frac{9}{\pi}} = \frac{3}{\sqrt{\pi}}$$

2) equation always true

$$A = \pi r^2$$

3) derivative

$$6 = \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

4) substitute...... umm....

$$6 = 2\pi\left(\frac{3}{\sqrt{\pi}}\right)\frac{dr}{dt} = \frac{6\pi}{\sqrt{\pi}}\frac{dr}{dt}$$

5) solve

$$\frac{dr}{dt} = \frac{6\sqrt{\pi}}{6\pi}\ mi/hr = \frac{1}{\sqrt{\pi}}\,\frac{mi}{hr}$$

$$\frac{\pi^{\frac{1}{2}}}{\pi^1} = \pi^{\frac{1}{2}-1} = \pi^{-\frac{1}{2}} = \frac{1}{\pi^{1/2}} = \frac{1}{\sqrt{\pi}}$$